

Dirac Sea and Observation Proposal

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Abstract

Assuming the Dirac sea is a physical reality, we have one imagination that a negative energy particle(s) in the sea and a usual positive energy particle(s) will form a neutral atom. The feature of the atom can be studied using the Schrödinger equation, especially for two body system. In this work, new physics meanings are given to the solution to the two body equation. It is noted it is hard to distinguish an atom consisting of a negative energy particle and a positive energy particle from a neutrino, but an atom X consisting of two negative energy particles and a nucleus with double charges can be observed in CLEOc, BESIII, Belle and BaBar for Charmonium or Bottomonium decays $[c\bar{c}]$ or $[b\bar{b}] \rightarrow X\pi^-\pi^-e^+e^+ + hc$ with X invisible.

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The Dirac sea^[1] is a theoretical model of the vacuum as an infinite sea of particles with negative energy. It was first postulated by the British physicist Paul Dirac in 1930 to explain the anomalous negative-energy quantum states predicted by the Dirac equation, an extension of the Schrödinger equation^[2] for relativistic electrons. The positron, the antimatter counterpart of the electron, was originally conceived of as a hole in the Dirac sea, well before its experimental discovery in 1932. Despite its success, the idea of the Dirac sea tends not to strike people as very elegant. The development of quantum field theory in the 1930s reformulate the Dirac equation in a way that treats the positron as a real particles rather than the absence of a particle and make the vacuum the state in which no particles exist instead of an infinite sea of particles. However, in this work, we assume Dirac's idea that there are particles with negative energy in the vacuum is completely correct, no matter they are particles or antiparticles and use the Schrödinger equation to investigate a possibility that a usual positive energy particle(s) and a negative energy particle(s) in the Dirac sea will form a neutral atom through electric-magnetic interaction.

The simplest scenario is a two body system. The positive energy particle n has the energy-momentum relation as

$$E_n = \sqrt{m_n^2 + p_n^2} = m_n + \frac{p_n^2}{2m_n} + \dots,$$

while the negative energy particle e in the sea has the energy-momentum relation as

$$E_e = -\sqrt{m_e^2 + p_e^2} = -m_e - \frac{p_e^2}{2m_e} + \dots$$

Though we don't know what meaning the negative energy is, we can assume the existence of negative energy particles is a physical reality. Time-independent Schrödinger equation to describe the two non-relativistic particles system is

$$E\psi(\mathbf{r}_n, \mathbf{r}_e) = -\frac{\hbar^2}{2m_n}\nabla_n^2\psi(\mathbf{r}_n, \mathbf{r}_e) + \frac{\hbar^2}{2m_e}\nabla_e^2\psi(\mathbf{r}_n, \mathbf{r}_e) + V(\mathbf{r}_n, \mathbf{r}_e)\psi(\mathbf{r}_n, \mathbf{r}_e),$$

$$V(\mathbf{r}_n, \mathbf{r}_e) = -\frac{Ze^2}{|\mathbf{r}_n - \mathbf{r}_e|}.$$

We can convert this two-body problem to an effective one-body problem by transforming from the laboratory system to the center of mass system

$$(\mathbf{r}_n, \mathbf{r}_e) \Rightarrow (\mathbf{R}, \mathbf{r}),$$

where $\mathbf{r} = \mathbf{r}_n - \mathbf{r}_e$, $M\mathbf{R} = m_n\mathbf{r}_n - m_e\mathbf{r}_e$, $M = m_n - m_e$. In this system, the wave function is separable $\psi(\mathbf{r}_n, \mathbf{r}_e) = \psi(\mathbf{R})\psi(\mathbf{r})$. We are only interested in $\psi(\mathbf{r})$. $\psi(\mathbf{r})$ satisfies the wave function

$$E\psi(\mathbf{r}) = \frac{\hbar^2}{2m}\nabla^2\psi(\mathbf{r}) + V(\mathbf{r})\psi(\mathbf{r}),$$

where the reduced mass $m_r = -m_n m_e / (m_n - m_e)$, $m = -m_r$

In spherical polar coordinates $\psi(\mathbf{r}) = R(r)\Theta(\theta)\Phi(\phi)$. In this work, only radial wave function $R(r)$ is worth discussing.

$$\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR}{dr} \right) - \left[\frac{2m}{\hbar^2} \left(E + \frac{Ze^2}{r} \right) + \frac{l(l+1)}{r^2} \right] R. \quad (1)$$

Let $R(r) = \frac{u(r)}{r}$, one has

$$\frac{d^2 u}{dr^2} - \left[\frac{2m}{\hbar^2} \left(E + \frac{Ze^2}{r} \right) + \frac{l(l+1)}{r^2} \right] u. \quad (2)$$

For a bound state, $E < 0$. We have

$$\frac{d^2 u}{dr^2} + \left[\frac{2m}{\hbar^2} \left(|E| - \frac{Ze^2}{r} \right) - \frac{l(l+1)}{r^2} \right] u. \quad (3)$$

If we transform $-\frac{Ze^2}{r} \Rightarrow \frac{Ze^2}{r}$, this equation is of the same form with the scattering equation with $m_r = m_n m_e / (m_n + m_e)$ and $E > 0$ for one positive energy particle scattered by the coulomb field of another positive energy particle

Let $\alpha = \left(\frac{8m|E|}{\hbar^2} \right)^{\frac{1}{2}}$, $\beta = \frac{Ze^2}{\hbar} \left(\frac{m}{2|E|} \right)^{\frac{1}{2}}$ with $|E| > 0$, $\rho = \alpha r$, one has

$$\frac{d^2 u}{d\rho^2} + \left[\frac{1}{4} - \frac{\beta}{\rho} - \frac{l(l+1)}{\rho^2} \right] u = 0.$$

One can't solve this straight away, but for very large ρ , the third and fourth terms are forced to zero because they go reciprocal with ρ . That leaves one with an asymptotic equation:

$$\frac{d^2 u_\infty}{d\rho^2} + \frac{1}{4} u_\infty = 0.$$

The solution is $u_\infty(\rho) = e^{\pm i\frac{\rho}{2}}$. Let $u_1(\rho) = e^{i(\frac{\rho}{2} + \phi_1)} f_1(\rho)$, $u_2(\rho) = e^{-i(\frac{\rho}{2} + \phi_2)} f_2(\rho)$, one has

$$f_1''(\rho) + i f_1'(\rho) + \left[-\frac{\beta}{\rho} - \frac{l(l+1)}{\rho^2} \right] f_1(\rho) = 0, \quad (4)$$

$$f_2''(\rho) - if_2'(\rho) + \left[-\frac{\beta}{\rho} - \frac{l(l+1)}{\rho^2} \right] f_2(\rho) = 0 \quad (5)$$

and

$$u(\rho) = au_1(\rho) + bu_2(\rho). \quad (6)$$

Let

$$f_1(\rho) = \sum_{n=0}^{\infty} a_n \rho^{n+s} \quad (7)$$

and substitute it to (4), one has the relation

$$a_{n+1} = \frac{\beta - i(n+s)}{(n+1+s)(n+s) - l(l+1)} a_n.$$

Due to $a_0 \neq 0$, one has

$$s(s-1) = l(l+1).$$

The solution is $s = -l$, $s = l+1$. If one take $s = -l = 0$, $a_1 \rightarrow \infty$. If one take $s = -l < 0$, the wave function $R(\rho) \rightarrow \infty$ when $\rho \rightarrow 0$. One should give up $s = -l$ and only reserves $s = l+1$. When $n \rightarrow \infty$, one has

$$\frac{a_{n+1}}{a_n} \rightarrow \frac{-i}{n}.$$

Note that the series

$$e^{-i\rho} = 1 + \frac{-i\rho}{1!} + \frac{(-i\rho)^2}{2!} + \dots + \frac{(-i\rho)^n}{n!} + \dots \quad (8)$$

The series (7) with $s = 0$ has the same behavior with the series (8) when $\rho \rightarrow \infty$.

If $l > 0$, $s = l+1 > 1$, $R(\rho) \rightarrow \infty$ when $\rho \rightarrow \infty$. One should only take $l = 0$.

One can calculate $f_2(\rho)$ in the same way. Let

$$f_2(\rho) = \sum_{n=0}^{\infty} b_n \rho^{n+s}, \quad (9)$$

$$b_{n+1} = \frac{\beta + i(n+s)}{(n+1+s)(n+s) - l(l+1)} b_n.$$

$f_2(\rho)$ is a complex conjugate of $f_1(\rho)$.

We have two independent real solutions. Let $fr(\rho)$ and $fi(\rho)$ stand for the real and imaginary parts of $f_1(\rho)$ respectively. The first solution is

$$\begin{aligned} R_1(\rho) &= \text{Re} \left[e^{i\left(\frac{\rho}{2} + \phi_1\right)} f_1(\rho)/\rho \right] \\ &= \cos(\rho + \phi_1) fr(\rho)/\rho - \sin(\rho + \phi_1) fi(\rho)/\rho. \end{aligned} \quad (10)$$

The second solution is

$$\begin{aligned} R_2(\rho) &= \text{Im} \left[e^{-i(\frac{\rho}{2} + \phi_2)} f_2(\rho)/\rho \right] \\ &= -\cos(\rho + \phi_2) fi(\rho)/\rho - \sin(\rho + \phi_2) fr(\rho)/\rho. \end{aligned} \quad (11)$$

Note that $R_1(\rho) \rightarrow \cos(\frac{\rho}{2} + \phi_1)$ and $R_2(\rho) \rightarrow \sin(\frac{\rho}{2} + \phi_2)$ when $\rho \rightarrow \infty$. Let $\phi_1 = 0$ and $\phi_2 = -\frac{1}{2}\pi$, $R(\rho) = a[R_1(\rho) + R_2(\rho)]$, one has the probability $\propto R^2(\rho) \rightarrow 0$ when $\rho \rightarrow \infty$. Finally,

$$R_1(\rho) = \cos(\rho) fr(\rho)/\rho - \sin(\rho) fi(\rho)/\rho. \quad (12)$$

$$R_2(\rho) = \sin(\rho) fi(\rho)/\rho + \cos(\rho) fr(\rho)/\rho. \quad (13)$$

Now, we turn to extract the physical information from the equations(1-9). The eq. (1) has the solution for the continuum energy $-\infty < E < 0$ and only the orbital quantum number $l = 0$. If the positive energy particle consists of $u\bar{d}$, and the negative energy particle is an electron, let $X(e^-)$ represent the atom of $u\bar{d}e^-$, the $X(e^-)$ has some feature of ν_e . The spin is $\frac{1}{2}$, The lepton number is 1, The lifetime is infinite. The mass tends to zero. The interaction is too weak. u has decays like $u \rightarrow u\bar{d}e^+e^- \rightarrow X(e^-)de^+$, and $u \rightarrow W^+d \rightarrow u\bar{d}d \rightarrow X(e^-)de^+$. If $X(e^-)$ is a real particle, one deduction is that if the positive energy particle consists of $u\bar{d}u\bar{d}$, and the negative energy particle is two electrons, let $X(e^-e^-)$ represent the atom of $u\bar{d}u\bar{d}e^-e^-$, the $X(e^-e^-)$ can be observed in CLEOC^[3], BESIII^[4], Belle^[5] and BaBar^[6] for Charmonium or Bottomonium decays $[c\bar{c}]$ or $[b\bar{b}] \rightarrow X(e^-e^-)\pi^-\pi^-e^+e^+ + hc$ with $X(e^-e^-)$ and hc invisible.

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